

May 2007: Math I

$$1) \begin{cases} 2x + 5y = 23 \\ 5x - 3y = 11 \end{cases}$$

$$\begin{array}{r} 10x + 25y = 115 \\ 10x - 6y = 22 \\ \hline 31y = 93 \\ y = 3 \end{array}$$

$$\begin{aligned} \Rightarrow 2x + 15 &= 23 \\ 2x &= 8 \\ x &= 4 \end{aligned}$$

$$x = 4, y = 3$$

$$\begin{cases} x^2 + 5\sqrt{y} = 23 \\ 5x^2 - 3\sqrt{y} = 11 \end{cases}$$

$$\begin{aligned} 31\sqrt{y} &= 93 \\ \sqrt{y} &= 3 \\ y &= 9 \end{aligned}$$

$$\begin{aligned} 2x^2 + 5(3) &= 23 \\ 2x^2 &= 8 \\ x^2 &= 4 \\ x &= \pm 2 \end{aligned}$$

$$x = \pm 2, y = 9$$

2) 3459, 9 = units digit
 0) $18^2 = 324$, 4 = units digit

• All digits ending in 8² have 4 as unit digit

2007¹, 7 = units digit
 2007², 9 = units digit since $7 \times 7 = 49$

7, 9, 3, 1,

2007³, $7 \times 7 \times 7 = 343$, 3 = units digit

2007⁴, $7 \times 7 \times 7 \times 7 = 2401$, 1 = units digit

2007⁵, $7 \times 7 \times 7 \times 7 \times 7 = 16807$ $\therefore 7 =$ units digit

2007⁶ $\Rightarrow 9 =$ units digit

2007⁸ $\Rightarrow 1 =$ units digit

2007⁷ $\Rightarrow 3 =$ units digit

$$\begin{array}{r} 62 \\ 62 \\ \hline 124 \\ 08 \\ \hline \end{array}$$
 units number is always this since we place a 0 underneath always. Just last 2 numbers multiplied together

b) $7, 9, 3, 1, 7, 9, 3, 1, 7, 9, 3, 1$

Sequence keeps repeating every 4

c) Power

1: 7

2: 9

3: 3

4: 1

5: 7

6: 9

Repeats every 4

i) $2007^{10} = \frac{10}{4} = 2 \quad 2007^2 = 9$ (9)

ii) $2007^{21} = 2$

$\frac{21}{4} = 5$ remainder 1
 $5 \times 4 = 20 \therefore \uparrow$

$= 2007^1 = 7$

iii) $2007^{43} \quad \frac{43}{4} = 10 \quad 4 \times 10 = 40 \therefore 3 = \text{remainder}$

$= 2007^3 = 3$

3) $S = \frac{ML^3}{8000W}$

a) $S = \frac{62(2.6)^3}{8000(0.7)} = 0.19459$

b) $0.4 = \frac{70(3.4)^3}{8000W}$

$3200W = 2751.28$

$W = 0.859775$

c) $1.1 = \frac{M(4)^3}{8000(0.5)} \Rightarrow 1.1 = \frac{64M}{4000} \Rightarrow 4400 = 64M$

$M = \frac{4400}{64} = 68.75$

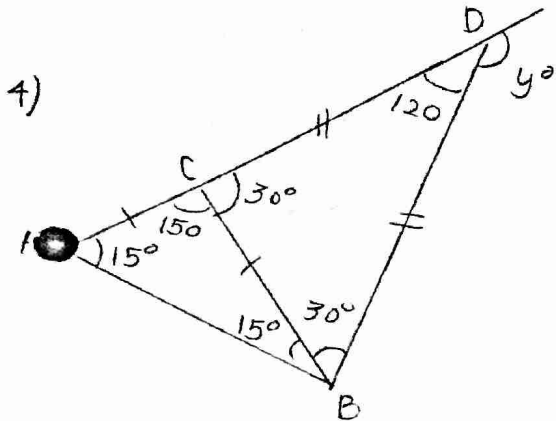
$$d) 0.8 = \frac{65L^3}{8000(0.3)}$$

$$0.8 = \frac{65L^3}{2400}$$

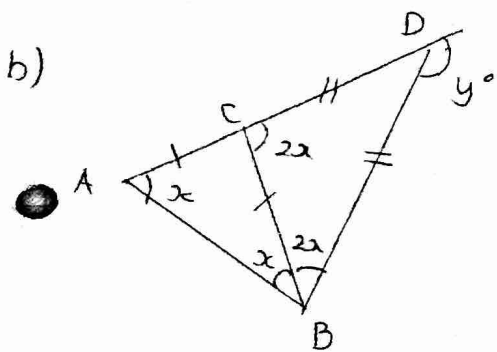
$$1920 = 65L^3$$

$$L^3 = \frac{1920}{65}$$

$$L = 3.09122$$



a) $y = 180 - 120 = 60$



$$\angle ACB = 180 - 2x$$

$$\angle BCD = 180 - (180 - 2x) = 2x$$

$$\angle BCD = 180 - 4x$$

$$y = 180 - (180 - 4x)$$

$$y = 4x$$

y is 4 times the size of x

c) As x gets close to 45, $\angle ACB$ gets close to 90, $\angle BCD$ gets close to 90 & $\angle CBD$ gets close to 45, since 2 isosceles triangles. D no longer makes triangle CDB. See diagram.

5) let $x = H$ of sheep
 $y = H$ of hens

$$\frac{4x+2y}{x+y} = 3.2 \quad \text{and} \quad \frac{4(x+1)+2(y-1)}{(x+1)+(y-1)} = 3.3$$

$$4x+2y = 3.2(x+y) \quad \text{and} \quad 4x+4+2y-2 = 3.3(x+y)$$

$$4x+2y = 3.2x+3.2y \quad 4x+2y+2 = 3.3x+3.3y$$

$$0.8x - 1.2y = 0 \quad 0.7x - 1.3y = -2$$

$$\begin{array}{r} 0.8x - 1.2y = 0 \\ 0.7x - 1.3y = -2 \end{array} \Rightarrow \begin{array}{r} \times 7 \\ 5.6x - 8.4y = 0 \\ \times 8 \\ 5.6x - 10.4y = -16 \end{array} \Rightarrow \begin{array}{r} 5.6x - 8.4y = 0 \\ 5.6x - 10.4y = -16 \\ \hline 2.0y = 16 \\ y = 8 \\ \Rightarrow x = 12 \end{array}$$

$$x = 12, y = 8$$

12 Sheep, 8 hens

6) a) $Bc^2 + 4^2 = 10^2$

$$Bc^2 = 84$$

$$Bc = 9.16515 \quad \text{or} \quad \sqrt{84}$$

$$8^2 + 9.16515^2 = x^2 \quad \text{or} \quad 8^2 + 84 = x^2$$

$$x^2 = 148$$

$$x = 12.1655$$

b) $x^2 + Bc^2 = 23^2$

$$Bc^2 = 23^2 - x^2$$

$$Bc = \sqrt{529 - x^2}$$

$$(2x)^2 + (\sqrt{529 - x^2})^2 = 31^2$$

$$4x^2 + 529 - x^2 = 961$$

$$3x^2 = 432$$

$$x^2 = \frac{432}{3}$$

$$x^2 = 144$$

$$x = 12$$

$$7) y = 2x\sqrt{64-x^2}$$

$$a) y = 2(4)\sqrt{64-16}$$

$$y = 8\sqrt{48} = 55.43$$

b) y = area of rectangle
 x = height of rectangle

When the height of the rectangle is 0 (ie $x=0$) there is no rectangle \therefore no area so $y=0$.

When the height of the rectangle is 8 it cannot fit inside the ^{semi}circle since the ^{semi}circle has radius 8. Therefore the rectangle cannot exist inside the circle \Rightarrow Area = 0 ($y=0$)

$$c) x=1 : y = 15.87$$

$$x=2 : y = 30.98$$

$$x=3 : y = 44.50$$

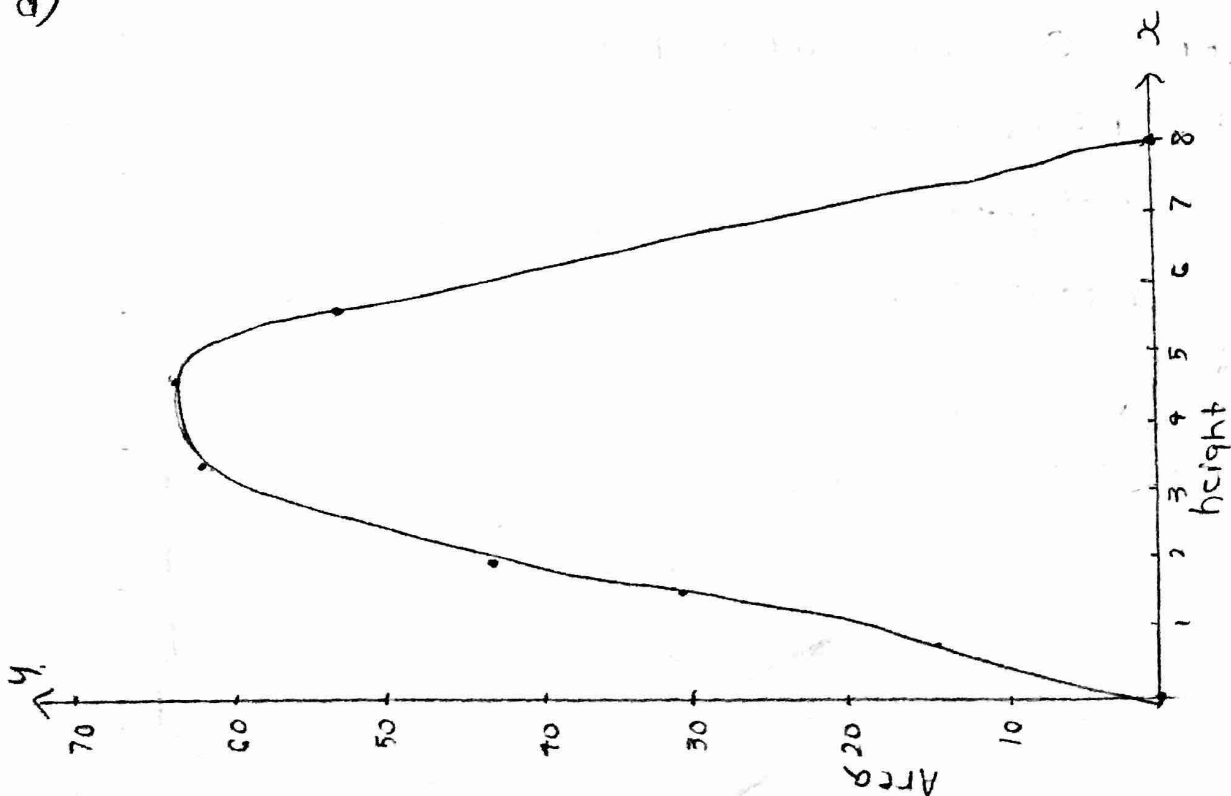
$$x=5 : y = 62.45$$

$$x=6 : y = 63.50$$

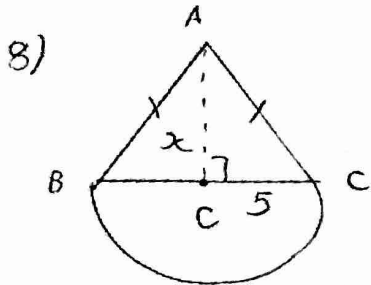
$$x=7 : y = 54.22$$

$$x=8 : y = 0$$

d)



- e) Max value of $y = 64$
 $x = 5.6568536, y = 64$
 Max Area = 64



a) Area $\Delta = \frac{1}{2}(10)(x) = 5x$
 Area $O = \pi r^2 = \pi(25) = 25\pi$

$$5x = 25\pi$$

$$x = \frac{25\pi}{5} = 5\pi$$

b) $AC^2 = x^2 + 5^2$

$$AC = \sqrt{x^2 + 25}$$

$$\text{Perimeter } \Delta ABC = 10 + \sqrt{x^2 + 25} + \sqrt{x^2 + 25}$$

$$\text{Perimeter } O = 2\pi r = 2\pi(5) = 10\pi$$

$$10\pi = 10 + 2\sqrt{x^2 + 25}$$

$$\sqrt{x^2 + 25} = \frac{10\pi - 10}{2}$$

$$x^2 + 25 = (5\pi - 5)^2$$

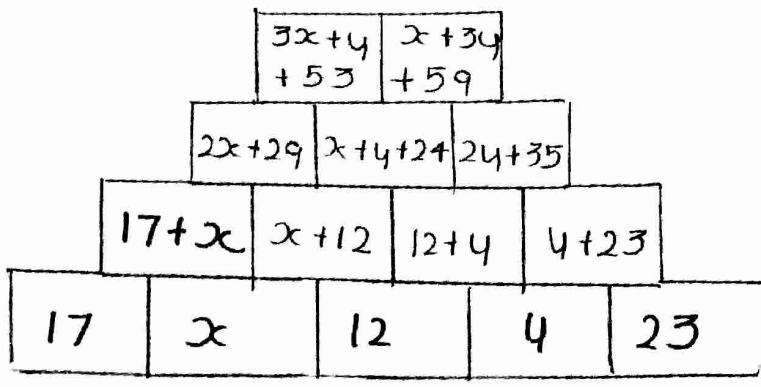
$$x^2 = (5\pi - 5)^2 - 25$$

$$x^2 = 89.660477$$

$$x = 9.469$$

q)

a)



b) $3x+y+53=79$
 $3x+y=26$

$$x+3y+59=73$$
$$x+3y=14$$

$3x+y=26$
 $x+3y=14$

$$\Rightarrow \begin{array}{l} 3x+y=26 \\ 3x+9y=42 \end{array}$$

$$-8y=-16$$

$$y=2$$

$$\Rightarrow x = \frac{26-2}{3} = 8$$

$$x=8, y=2$$

c) $3x+y+53=73$
 $3x+y=20$

$$x+3y+59=79$$
$$x+3y=20$$

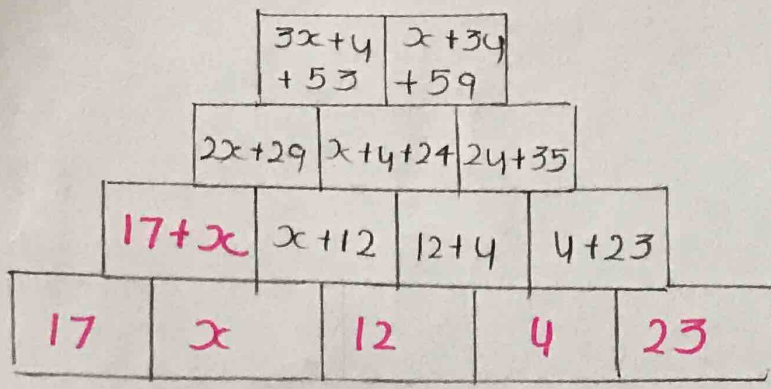
$$x=5$$

$$y=5$$

Value of x and y will be the same

q)

a)



b) $3x+y+53=79$
 $3x+y=26$

$x+3y+59=73$
 $x+3y=14$

$3x+y=26$
 $x+3y=14$

$\Rightarrow 3x+y=26$
 $3x+9y=42$

$-8y=-16$

$y=2$

$\Rightarrow x = \frac{26-2}{3} = 8$

$x=8, y=2$

c) $3x+y+53=73$
 $3x+y=20$

$x+3y+59=79$
 $x+3y=20$

$x=5$

$y=5$

Value of x and y will be the same